I) 1) 10 possibilités pr chaque $= 10^4$

2) 1 possi pr le 1^o, 10 pr les 3 autres : $1 \cdot 10^3 = 10^3$

3) 4 arrangements : $\dfrac{10!}{(10-4)!} = 10 \times 9 \times 8 \times 7$

4) tous impairs : 5 possibilités pr chaque : 5^4

dc tous-impair $= 10^4 - 5^4 = 5^4(2^4-1) = 5^4 \times 15 = 3 \times 5^5$

5) choix du pair : $5 = \binom{5}{1}$
position du pair : $\binom{4}{1} = 4$
choix des 3 autres parmi les impairs : 5^3
$\Big\}$ $4 \times 5 \times 5^3 = 4 \times 5^4$

II) 1) $D = \mathbb{R}^2$

2) $\dfrac{\partial P}{\partial y} = \dfrac{-2xe^y}{(1+x^2)^2}$; $\dfrac{\partial Q}{\partial x} = \dfrac{-e^y}{(1+x^2)^2}\left[2x(1+x^2) - x^2(2x)\right] = \dfrac{\partial P}{\partial y}$

elle est fermée

3) a) fermée sur 1 ouvert étoilé $(\mathbb{R}^2)$, dc d'après Th de P, elle est exacte.

b) on cherche f C^1 tq $\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{-2x(1+e^y)}{(1+x^2)^2} \\ \dfrac{\partial f}{\partial y} = \dfrac{-x^2e^y}{1+x^2} \end{cases}$

on intègre la 2^{nd} selon y : $f(x,y) = \dfrac{-x^2e^y}{1+x^2} + h(x)$

dérive / x : $\dfrac{-2x(1+e^y)}{(1+x^2)^2} = \dfrac{\partial f}{\partial x} = h'(x) - \dfrac{e^y}{(1+x^2)^2}\left(2x(1+x^2) - x^2(2x)\right)$

dc $h'(x) = \dfrac{-2x}{(1+x^2)^2}$ on intègre : $h(x) = \dfrac{1}{1+x^2} + c$

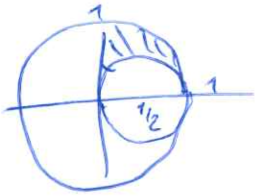
dc $f(x,y) = \dfrac{1-x^2e^y}{1+x^2}$

4) $\gamma(0) = (1+\cos(0); 1-\sin(0)) = (2;1)$ $f(2,1) = \dfrac{1-4e}{1+4} = \dfrac{1-4e}{5}$

$\gamma(\pi) = (1-1; 1-0) = (0,1)$ $f(0,1) = 1$

dc $\displaystyle\int_\gamma \omega = f(0,1) - f(2,1) = 1 - \frac{1}{5} + \frac{4e}{5} = \boxed{\frac{4}{5}(1+e)}$

III 1°)



II

2°) en polaire : $x \geq 0, y \geq 0 \iff \theta \in [0, \pi/2]$

$x \leq x^2+y^2 \iff \cos(\theta) \leq \rho$

$x^2+y^2 \leq 1 \iff \rho \leq 1$

d'où $I = \displaystyle\int_{\theta=0}^{\pi/2} \int_{\rho=\cos(\theta)}^{1} \rho(\cos(\theta)+\sin(\theta))\rho \, d\rho \, d\theta$

$= \displaystyle\int_0^{\pi/2} (\cos(\theta)+\sin(\theta)) \left[\frac{\rho^3}{3}\right]_{\cos(\theta)}^{1} d\theta$

$I = \dfrac{1}{3}\displaystyle\int_0^{\pi/2} (\cos(\theta)+\sin(\theta))(1-\cos^3(\theta))\,d\theta = \dfrac{1}{3}\int_0^{\pi/2} \cos(\theta)+\sin(\theta)-\cos^4(\theta)-\sin(\theta)\cos^3(\theta)\,d\theta$

après linéarisation : $\cos^4(\theta) = 3/8 + \dfrac{\cos(2\theta)}{2} + \dfrac{\cos(4\theta)}{8}$

$I = \dfrac{1}{3}\left[\sin(\theta) - \cos(\theta) - \dfrac{3\theta}{8} - \dfrac{\sin(2\theta)}{4} - \dfrac{\sin(4\theta)}{32} + \dfrac{\cos^4(\theta)}{4}\right]_0^{\pi/2}$

$= 1/3\left[1 - 0 - \dfrac{3\pi}{16} - 0 - 0 + 0 - 0 + 1 + 0 + 0 + 0 - \dfrac{1}{4}\right]$

$= 1/3\left[\dfrac{7}{4} - 3\pi/16\right] = \dfrac{7}{12} - \dfrac{\pi}{16}$

3°) on pose $\dfrac{\partial P}{\partial y} = 0$, $\dfrac{\partial Q}{\partial x} = x+y$ d'où $Q(x,y) = \dfrac{x^2}{2} + yx$

d'où $I = \displaystyle\int_\gamma \left(\frac{x^2}{2} + xy\right) dy$

$\mathcal{C}_1 : \begin{cases} x = \cos\theta & dx = -\sin\theta\, d\theta \\ y = \sin\theta & dy = \cos\theta\, d\theta \end{cases}$ θ de 0 à $\pi/2$

$\displaystyle\int_{\mathcal{C}_1} \omega = \int_0^{\pi/2} \left(\frac{\cos^2\theta}{2} + \cos(\theta)\sin(\theta)\right)\cos(\theta)\,d\theta = \int_0^{\pi/2}\left(\frac{1-\sin^2\theta}{2} + \cos(\theta)\sin\theta\right)\cos\theta\,d\theta$

$= \left[\dfrac{\sin(\theta)}{2} - \dfrac{\sin^3\theta}{6} - \dfrac{\cos^3\theta}{3}\right]_0^{\pi/2} = \dfrac{1}{2} - \dfrac{1}{6} - 0 - 0 + 0 + \dfrac{1}{3} = 2/3$

III

Sur $\mathcal{C}_2$: $\begin{cases} x=0 & dx=0 \\ y=t & dy=dt \end{cases}$ t de ~~p~~ à 0 $\quad \omega=0$ donc $\int_{\mathcal{C}_2}=0$

Sur $\mathcal{C}_3$: $\begin{cases} x=\frac{1}{2}(1+\cos(\theta)) \\ y=\frac{1}{2}\sin(\theta) \end{cases}$ $\begin{cases} dx=-\frac{1}{2}\sin(\theta)d\theta \\ dy=\frac{\cos(\theta)}{2}d\theta \end{cases}$ θ de π à 0

$$\int_{\mathcal{C}_3}\omega=\int_{\pi}^{0}\left(\frac{1}{8}(1+\cos(\theta))^2+\frac{1}{4}\sin(\theta)(1+\cos(\theta))\right)\frac{\cos(\theta)}{2}d\theta$$

$$=\frac{1}{16}\int_{\pi}^{0}\cos(\theta)(1+2\cos(\theta)+1-\sin^2(\theta))d\theta+\frac{1}{8}\int_{\pi}^{0}\cos(\theta)\sin(\theta)+\sin(\theta)\cos^2(\theta)d\theta$$

$$=\frac{1}{16}\Big[\cancel{\cos(\theta)}\,2\sin(\theta)+\sin(2\theta)-\theta-\frac{\sin^3(\theta)}{3}\Big]_{\pi}^{0}+\frac{1}{8}\Big[\frac{\sin^2(\theta)}{2}-\frac{\cos^3(\theta)}{3}\Big]_{\pi}^{0}$$

$$=-\pi/16+1/8(-1/3-1/3)=-\pi/16-1/12$$

au total : $2/3-\pi/16-1/12=7/12-\pi/16$

IV 1) a) $\frac{\partial P}{\partial y}=1=\frac{\partial Q}{\partial x}$ donc ω fermée sur $\mathbb{R}^2$ étoilé donc exacte

Γ = lacet donc $\int_\Gamma\omega=0$

b) $\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dy\,dx=\iint_D 0\,dx\,dy=0$

2) a) Sur $[OA]$ $\begin{cases} x=t & dx=dt \\ y=0 & dy=0 \end{cases}$ $\quad\omega=0$ donc $\int_{[OA]}\omega=0$

Sur $[AB]$: $\begin{cases} x=1 & dx=0 \\ y=t & dy=dt \end{cases}$ $\quad\omega=t^3dt$ $\int_{[AB]}t^3dt=\left[\frac{t^4}{4}\right]_0^2=4$

Sur $[BC]$: $\begin{cases} x=t/2 & dx=dt/2 \\ y=t & dy=dt \end{cases}$ t de 2 à 0

$$\int_{[BC]}\omega=\int_2^0\left(\frac{t^2}{4}+\frac{t^5}{4}\right)dt=\left[\frac{t^3}{12}+\frac{t^6}{24}\right]_2^0=-\frac{8}{24}[2+8]$$

$$=-\frac{10}{3}$$

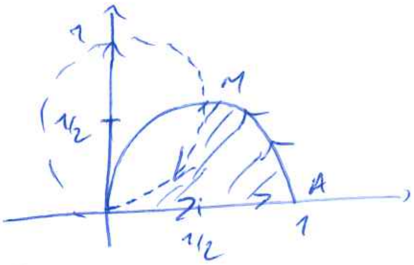
total : $4-10/3=2/3$

5) $\frac{\partial P}{\partial y} = x \qquad \frac{\partial Q}{\partial x} = 2xy^2 \qquad I = \iint_T x(2y^2-1)\,dx\,dy = \int_{x=0}^{1} x\left(\int_{y=0}^{2x} (2y^2-1)\,dy\right)dx$ IV

$$I = \int_0^1 x\left[\frac{2y^3}{3} - y\right]_0^{2x} dx = \int_0^1 x\left(\frac{2}{3}\cdot 8x^3 - 2x\right)dx = \left[\frac{4}{3}x^4 - \frac{2x^2}{3}\right]_0^1$$

$$= 4/3 - 2/3 = 2/3$$

V) 1.)



2.) $M(x,y) \in C_1 \cap C_2 \Rightarrow x^2+y^2-x = x^2+y^2-y \Rightarrow x=y$

ds ce cas $x^2+y^2-x=0 \Leftrightarrow 2x^2-x=0 \Leftrightarrow x=0$ ou $1/2$

dc $C_1 \cap C_2 = \{0\} \cup \{M\}$ avec $M(1/2\,;1/2)$

3.) $C_1 = \{(x,y) \in \mathbb{R}^2 /\ (x-1/2)^2 + y^2 = (1/2)^2\ ;\ y \geq 0\}$

$C_2 = \{(x,y) \in \mathbb{R}^2 /\ x^2 + (y-1/2)^2 = (1/2)^2\ ;\ x \geq 0\}$

D'où la paramétrisation proposée (quart de cercle)

4.) $\int_{\partial^+ \Omega} w = \int_{[OA]} w + \int_{C_1^1} w + \int_{C_2^2} w$

Sur $[OA]$: $y=0$ dc $w=0$ dc $\int_{[OA]} = 0$

Sur C_1^1 : $w = \left(-\left(\frac{1}{2}(1+\cos(t))\right)^2 \frac{1}{2}\sin(t)\left(-\frac{1}{2}\sin(t)\right) + \frac{1}{4}\sin^2(t)\frac{1}{2}(1+\cos(t))\frac{1}{2}\cos(t)\right)dt$

$$w = \frac{1}{16}\left[\cos(t)\sin^2(t) + \cos^2(t)\sin^2(t) + \sin^2(t)(1+2\cos(t)+\cos^2(t))\right]dt$$

$$= \frac{1}{16}\left[2\cos^2(t)\sin^2(t) + 3\cos(t)\sin^2(t) + \sin^2(t)\right]dt$$

$$= \frac{1}{16}\left[2\left(\frac{\sin(2t)}{2}\right)^2 + 3\cos(t)\sin^2(t) + \frac{1-\cos(2t)}{2}\right]dt$$

$$= \frac{1}{16}\left[\frac{1}{2}\left(\frac{1-\cos(4t)}{2}\right) + 3\cos(t)\sin^2(t) + \frac{1}{2} - \frac{\cos(2t)}{2}\right]dt$$

dc $\int_{C_1^1} w = \frac{1}{16}\int_0^{\pi/2} [\quad] dt = \frac{1}{16}\left[\frac{3t}{4} - \frac{\sin(4t)}{16} + \sin^3(t) - \frac{\sin(2t)}{4}\right]_0^{\pi/2}$

$= \frac{1}{16}\left[\frac{3\pi}{8} + 1\right] = \frac{1}{16} + \frac{3\pi}{128}$

Sur C_2^2 : $w = \left[-\frac{1}{4}\cos^2(t)\frac{1}{2}(1+\sin(t))\left(-\frac{1}{2}\sin(t)\right) + \frac{1}{4}(1+\sin(t))^2\frac{1}{2}\cos(t)\frac{1}{2}\cos(t)\right]dt$

$= \frac{dt}{16}\left[\sin(t)\cos^2(t) + \sin^2(t)\cos^2(t) + \cos^2(t)(1+2\sin(t)+\sin^2(t))\right]$

$= \frac{dt}{16}\left[3\sin(t)\cos^2(t) + \cos^2(t) + 2\sin^2(t)\cos^2(t)\right]$

$= \frac{dt}{16}\left[3\sin(t)\cos^2(t) + \left(\frac{1+\cos(2t)}{2}\right) + \frac{1}{4}(1-\cos(4t))\right]$

dc $\int_{C_2^2} w = \frac{1}{16}\left[-\cos^3(t) + \frac{3t}{4} + \frac{\sin(2t)}{4} - \frac{\sin(4t)}{16}\right]_0^{-\pi/2}$

$= \frac{1}{16}\left[0 - \frac{3\pi}{8} + 0 - 0 + 1 - 0 + 0\right] = \frac{1}{16}\left[1 - \frac{3\pi}{8}\right]$

au final : $\frac{1}{16}\left[1 + \frac{3\pi}{8} + 1 - \frac{3\pi}{8}\right] = \frac{2}{16} = \frac{1}{8}$

5) par G.R : $\iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \iint_\Omega (y^2 + x^2)\, dx\,dy.$

6) $y \geq 0$ dc $\theta \in [0\ \pi]$

$\rho\sin\theta \leq \rho^2 \leq \rho\cos\theta$ dc $\sin(\theta) \leq \rho \leq \cos(\theta)$

dc $\sin(\theta) \leq \cos(\theta)$ d'où $0 \leq \theta \leq \pi/4$

D'où le domaine.

7) a) $\cos^4(\theta) - \sin^4(\theta) = (\cos^2(\theta) - \sin^2(\theta))(\cos^2(\theta) + \sin^2(\theta)) = \cos^2(\theta) - \sin^2(\theta)$
$= \cos(2\theta)$

b) $I = \int_{\theta=0}^{\pi/4}\int_{\rho=\sin\theta}^{\cos\theta} \rho^2\,\rho\,d\rho\,d\theta = \frac{1}{4}\int_0^{\pi/4}(\cos^4(\theta) - \sin^4(\theta))\,d\theta$

$= \frac{1}{4}\int_0^{\pi/4}\cos(2\theta)\,d\theta = \frac{1}{8}\left[\sin(2\theta)\right]_0^{\pi/4} = 1/8$