Compressive Sensing Ultrasound Imaging using Overcomplete Dictionaries

Conference Paper · July 2013

DOI: 10.1109/ULTSYM.2013.0012

4 authors:


Oana Lorintiu
Cytoo


Hervé Liebgott
Creatis Medical Imaging Research Center


Olivier Bernard
Institut National des Sciences Appliquées de Lyon


Denis Friboulet
Creatis Medical Imaging Research Center

Compressive Sensing Ultrasound Imaging using Overcomplete Dictionaries

Oana Lorintiu, Hervé Liebgott, Olivier Bernard, Denis Friboulet

Université de Lyon, CREATIS ; CNRS UMR5220 ; Inserm U1044 ; INSA-Lyon ; Université Lyon 1, France

***Abstract*—The application of CS to medical ultrasound (US) imaging is a very recent field and the few existing studies mostly focus on fixed sparsifying transforms. In contrast to previous work, we propose a new approach based on the use of learned overcomplete dictionaries. Such dictionaries allow for much sparser representations of the signals since they are optimized for a particular class of images such as US images. In this study, the dictionary was learned using the K-SVD algorithm on patches extracted from the image to be reconstructed for an initial validation and on a training set of images afterwards. Experiments were performed on experimental beamformed RF data from a cyst numerical phantom. CS reconstruction was performed by removing 25% to 75% of the original samples according to a uniform law. Reconstructions using a K-SVD dictionary previously trained dictionary on experimental US images show minimal information loss, thus showing the potential of the overcomplete dictionaries.**

***Index Terms*—Compressive sensing, sparse representation, overcomplete dictionaries, ultrasound imaging.**

I. Introduction

In ultrasound (US) imaging the amount of data acquired to reconstruct the echographic image can be limiting for real time applications and can lead to data storage issues. In this context, the recently introduced Compressed Sensing (CS) theory offers the perspective of reducing the amount of data acquired and speeding up the acquisitions. CS is based on the idea that it is possible, under certain assumptions, to recover a signal sampled below the Nyquist sampling limit [1]–[3]. Its application to medical ultrasound imaging is promising due to the novelty of the technique and to the fact that it has led to only few studies [4].

In contrast to previous work, we propose a new approach based on the use of learned overcomplete dictionaries. Our choice is motivated by the fact that the existing studies mostly focus on fixed sparsifying transforms [5]–[10]. Since sparsity is an important prerequisite to achieve accurate CS reconstruction, we have chosen to investigate learned overcomplete data-driven dictionaries. Such dictionaries allow for much sparser representations of the signals since they are optimized for a particular class of images such as US images.

The main purpose of this paper is to show the feasibility of reconstructing high quality US images from less elements or sampled data using CS theory. First, the principle of CS is briefly recalled. Second, the dictionary learning algorithm is briefly described. Section IV presents the application of the proposed approach to experimental data and section V presents the results.

II. Compressive sensing theory

Compressive sensing (CS) [2] allows the reconstruction of a signal $x \in \mathbb{R}^n$ from a linear combination of a small number of random measurements $y \in \mathbb{R}^m, m < n$. In a general setting, the measurements y may be acquired in the so-called "sensing basis" Φ , which depends on the acquisition device. As examples, in MRI, Φ is the Fourier basis and in ultrasound, Φ simply consists in the usual delta functions. We then have:

$$y = R\Phi x \tag{1}$$

where $R\Phi$ is thus a $m \times n$ matrix. The columns of R have an entry one at random positions and zero elsewhere, thereby modeling the random selection of the measurements.

The CS theory assumes that x has a sparse representation in some model basis Ψ, which can be an orthonormal basis, a frame or an overcomplete dictionary, such that:

$$x = \Psi v \tag{2}$$

where v has only $s < m < n$ non zero coefficients. The signal v is called s-sparse. CS theory shows that this sparsity allows an exact recovering of v with overwhelming probability for a certain class of matrices $\Phi\Psi$ [3]. In particular, the sensing basis Φ has to be incoherent with the model basis Ψ [11], which is ensured by the randomness of the non-zero components of $R\Phi$. Finally, the problem can be written as follows:

$$y = R\Phi\Psi v = Av \tag{3}$$

where A is a $m \times n$ full rank matrix (i.e. the m rows of A are independent).

In these settings, the CS problem thus amounts to solve (3) for v , under the constraint that v is sparse. Once v is estimated, the signal x , can then be computed from (2).

For matrices A with a specified isometry constant of the so-called "restricted isometry property" (RIP), Candès et al. [3] showed that the CS problem may be solved through the following l_0-minimization problem P_0:

$$P_0 : \quad \hat{v} = arg \min_{v \in \mathbb{R}^n} \|v\|_0 \quad \text{subject to} \quad y = Av \tag{4}$$

where the l_0 quasi-norm of v is $\|v\|_0 = |\{i, v_i \neq 0\}|$.

P_0 implies that from all the possible solutions of (3), we seek the sparsest one. In general, solving (4) is NP-hard. Suboptimal greedy algorithm attempt to solve this problem by successively adding nonzero components to a sparse approximation of v .

By imposing a more restrictive bound on the isometry constant, the sparsest solution $\hat{x}$ of (3) can be found by solving the following basis pursuit (BP) problem P_1 [12]:

$$P_1 : \quad \hat{x} = arg \min_{v \in \mathbb{R}^n} \|v\|_1 \quad \text{subject to} \quad y = Av \tag{5}$$

where the l_1 norm of v is $\|v\|_1 = \sum_{i=1}^{n} |v_i|$. The $l_0 - l_1$ equivalence, using the RIP, was presented by Candès in [12] (see also [3]).

The framework described above assumes that we are given exact samples of the signal to be recovered. This is seldom the case in practice, since the measurements are very often corrupted by noise. For reconstruction of measurements with additive noise, we have:

$$y = Av + e \tag{6}$$

where e represents a noise term of bounded energy $\|e\|_2 \leq \varepsilon$. P_1 can be recast as [12]:

$$P_2 : \quad \hat{x} = arg \min_{v \in \mathbb{R}^n} \|v\|_1 \quad \text{subject to} \quad \|y - Av\|_2 \leq \varepsilon \tag{7}$$

In practical applications the signal is generally not exactly sparse but most of its coefficients in (2) are small. When signal coefficients v decays exponentially in absolute value, the signal is called compressible. The solution found by P1 (5) or P2 (7) gives the approximation of v by keeping its largest entries.

III. Learning overcomplete dictionaries

The CS theory assumes that x has a sparse representation in some model basis which, in this study, will be an overcomplete dictionary rather than a fixed basis. Such dictionaries allow for much sparser representations of the signals since they are optimized for a family of signals that are of interest such as US images.

Dictionary learning uses a set of training samples $Y = \{y_i\}_{i=1}^{N}$ to find D an optimal dictionary that will best sparsify them. This can be formulated as the following minimization problem:

$$\min_{D,X} \|Y - DX\|_F^2 \quad \text{s.t.} \quad \forall 1 \leq i \leq N, \|x_i\|_0 \leq T_0 \tag{8}$$

where T_0 is the number of non-zero entries, which is expected to be very small, Y contains all the training samples as columns and X contains the corresponding coefficients.

Solving the dictionary learning problem is also NP-hard and numerous algorithms have been proposed, the literature on this topic being vast and fast growing. Tosic et al. give a review of these methods in [13]. In this study we have chosen the K-SVD algorithm due to its efficiency and ease of implementation.

The K-SVD method solves iteratively the optimization problem using two steps: the sparse-coding step and the dictionary update step. In the sparse-coding stage we assume the knowledge of D and we find X using any pursuit algorithm. The minimization problem (8) can be decomposed as following:

$$\min_{x_i} \|y_i - Dx_i\|_2^2 \quad \text{s.t.} \quad \|x_i\|_0 \leq T_0, \text{ for } i = 1, 2, .., N \tag{9}$$

and is usually solved using OMP. In the second stage, the algorithm updates D one atom at a time and its corresponding non-zero coefficients in X, thus providing an update of both D and X. The algorithm iterates between the two steps until convergence. K-SVD algorithm and variations can be seen in detail in [14], [15].

IV. Application to US images

A. Experimental phantom acquisition setup

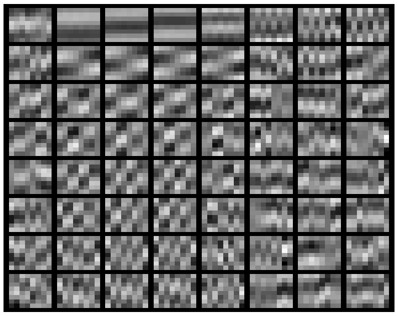
The experimental channel RF data were acquired using the Ultrasonix MDP research platform equipped with the parallel channel acquisition system SonixDaq. The data consisted in 128 single element received signals of a linear L14-5W/60 Prosonic (Korea) probe. The central frequency of the probe was 7 MHz and the signals were collected using a 40-MHz sampling rate. The medium imaged was a general-purpose ultrasound phantom CIRS Model 054GS. The beamformed data are calculated using the delay and sum beamformer using a constant Hanning apodization over the 128 receive elements.

B. Reconstruction scheme

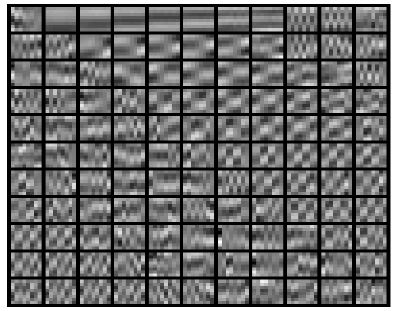
The experimental beamformed RF signals produced either by the experimental acquisition on phantom data were gathered and formed the original data, x. These original beamformed images were then subsampled by removing varying amounts of samples. The spatial position of the removed samples was selected according to a uniform random law. CS reconstruction was then performed from the subsampled data sets where 25% to 75% of the original samples were removed.

CS reconstruction using an overcomplete dictionary was performed using a block-wise approach. Let x_p of $\mathbb{R}^n, n < N$, be a patch of an image x and D of $\mathbb{R}^{n \times K}$ be an overcomplete dictionary, with $n < K$, such that $x_p = Dv_p$. Application of CS implies learning D such that v_p is a sparse representation of the patch p in D. We will then be able to recover the original image patches from the linear measurements y. The implementation of the approximate K-SVD presented by Rubinstein [16] were used with the improvements proposed in [15] for the learning of the overcomplete dictionary and the OMP algorithm was used for the block-wise CS reconstruction.

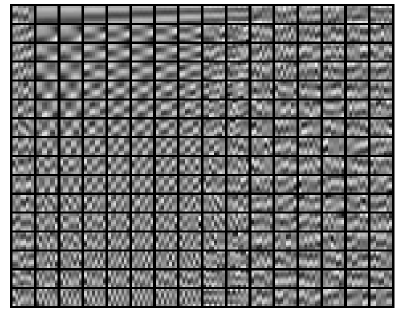
The accuracy of the results was quantified by comparing the CS reconstruction to the original data through the normalized root mean square error (NRMSE). Besides the K-SVD dictionary, we include two other bases in our experiments



(a) 64 8×8 atoms



(b) 128 8×8 atoms



(c) 256 8×8 atoms

Fig. 1. Examples of dictionaries learned using K-SVD on patches extracted from the original beamformed image. Different setup are illustrated, with the number of atoms varying.

for comparison: the Fourier basis and the Daubechies db5 wavelets.

V. Experimental results

A. Ultrasound phantom images

1) Dictionary learning: The training data consisted of 8×8 patches extracted from the original beamformed RF image and the testing data consisted of the very same image. We use typical values for the algorithm parameters: sparsity $T_0 = 8$; patch dimension $n = 64$ (i.e. 8×8); and overcompleteness of $K = 4n$ (i.e. $4 \times 64 = 256$). Figure 1 shows examples of such learned dictionaries for different numbers of atoms.

2) Sparsity of US images: In order to apply the CS theory we made the assumption of sparsity in a given basis Ψ. The concept of sparsity can be illustrated by plotting the sparse coefficients in a given basis in order of magnitude. If they decay rapidly, then the compressed signal containing the s largest coefficients will be close to the original signal and this percentage of transform coefficients will be sufficient to get reliable reconstructions. If in addition to sparsity we satisfy the hypothesis of incoherence then the original signal can be recovered exactly with overwhelming probability.

Figure 2 illustrates the concept of sparsity in the domain of three transforms: Daubechies wavelets, Fourier basis and K-SVD dictionary. The transform coefficients decay very rapidly, indicating a sparse representation of the image. Because the sparse representation in the K-SVD dictionary decays the fastest in comparison to the other transforms, it is possible to have a higher quality reconstruction using only 25% of its largest coefficients.

3) Reconstruction results: Figure 3 shows the reconstruction NRMSE error as a function of the subsampling rate and for each of the transforms used for reconstruction. It can be observed that the error increases with the number of removed samples, whatever the reconstruction basis. The error corresponding to the wavelets takes the largest values, and the error associated to the Fourier basis shows intermediate values. The K-SVD dictionary gives the smallest error, whatever the subsampling rate.

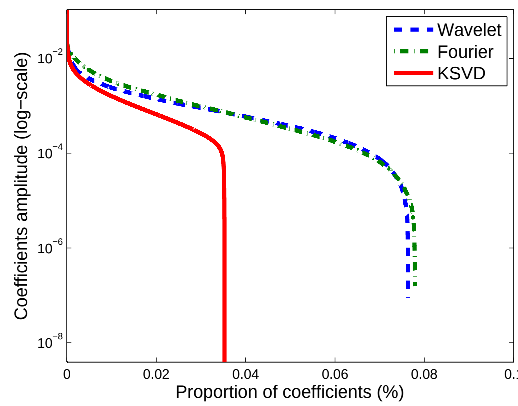


Fig. 2. Ordered relative values of the K-SVD dictionary, Fourier and wavelet transform coefficients, for an US image.

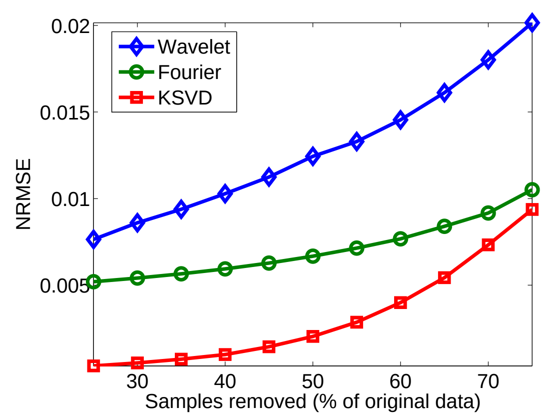


Fig. 3. NRMSE as a function of the number of removed samples. The error is computed on the RF beamformed images after CS reconstruction using K-SVD dictionary, Fourier and wavelets.

Figure 4 shows the error images of the CS reconstruction using wavelets, Fourier and K-SVD dictionary. In this example, the reconstructions have been performed after removing 75% of the original data. These images suggest that the higher errors are associated with the wavelet and Fourier reconstructions. Whatever the reconstruction sparsifying basis, the higher error values are located in the hyperechoic cyst and the different high-intensity scatterers, areas associated to

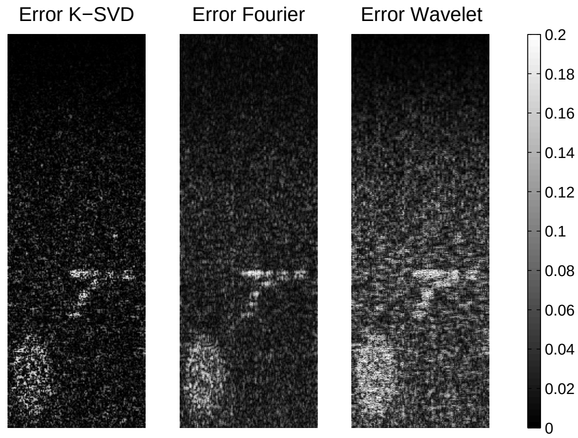


Fig. 4. Absolute error between original image and reconstructed RF data using K-SVD dictionary, Fourier and wavelets. Data were reconstructed using 75% subsampling. The error images were scaled in the interval [0, 0.2] for better visibility.

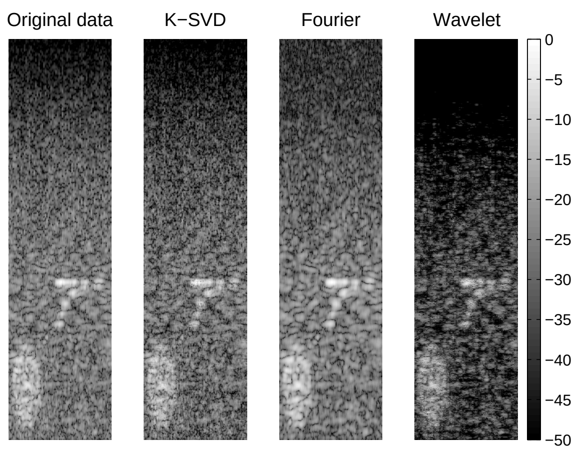


Fig. 5. Log-envelope beamformed images computed from the original data and from CS reconstruction using K-SVD dictionary, Fourier and wavelets. Data were reconstructed using 75% subsampling.

large signal amplitude.

Figure 5 shows the log-envelope images corresponding to the reconstructed RF data, for better visibility and figure 6 the corresponding error images. Due to the log operation, which performs amplitude compression, the error is now less localized and appears to be more evenly spread over the image. The K-SVD dictionary reliably recovers the large scale hyperechoic structures as well as the speckle, whereas the Fourier basis tends to smooth these structures and the wavelets completely fail to recover the top part of the image.

VI. Conclusion

This study demonstrates the high potential of learned overcomplete dictionaries for CS in US imaging. Experiments performed on experimental RF data with the K-SVD based CS reconstruction using only 25% of the initial samples resulted in US images close to the original, with minimal loss of information. Thereby, compressive sensing (CS) reconstruction with overcomplete dictionaries is feasible in medical ultrasound.

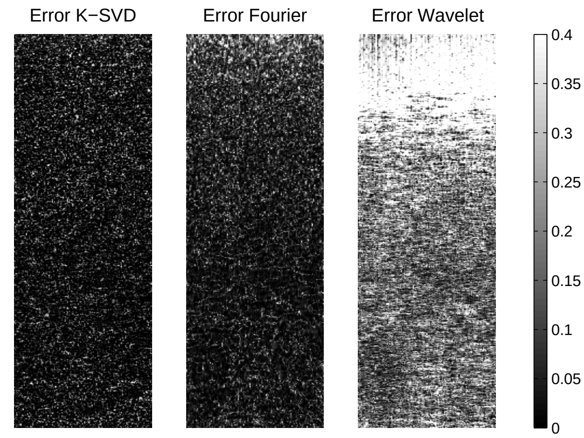


Fig. 6. Absolute error between beamformed log-envelope image computed from the original data and CS reconstruction using K-SVD dictionary, Fourier and wavelets. Data were reconstructed using 75% subsampling. The error images were scaled in the interval [0, 0.4] for better visibility.

Acknowledgment

This work was performed within the framework of the LABEX PRIMES (ANR-11-LABX-0063) of Université de Lyon, within the program "Investissements d'Avenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR).

References

[1] D. Donoho, "Compressed sensing," *Information Theory, IEEE Transactions on*, vol. 52, no. 4, pp. 1289–1306, 2006.
[2] E. Candes and M. Wakin, "An introduction to compressive sampling," *Signal Processing Magazine, IEEE*, vol. 25, no. 2, pp. 21–30, 2008.
[3] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *Information Theory, IEEE Transactions on*, vol. 52, no. 2, pp. 489–509, 2006.
[4] H. Liebgott, A. Basarab, D. Kouamé, O. Bernard, and D. Friboulet, "Compressive sensing in medical ultrasound," *IEEE International Ultrasonics Symposium*, pp. 1938–1943, 2012.
[5] H. Liebgott, R. Prost, and D. Friboulet, "Pre-beamformed rf signal reconstruction in medical ultrasound using compressive sensing," *Ultrasonics*, vol. 53, no. 2, 2013.
[6] J. Richy, H. Liebgott, R. Prost, and D. Friboulet, "Blood velocity estimation using compressed sensing," pp. 1427–1430, 2011.
[7] A. Achim, B. Buxton, G. Tzagkarakis, and P. Tsakalides, "Compressive sensing for ultrasound rf echoes using a-stable distributions," pp. 4304–4307, 2010.
[8] N. Wagner, Y. C. Eldar, A. Feuer, and Z. Friedman, "Compressed beamforming applied to b-mode ultrasound imaging," *CoRR*, vol. abs/1201.1200, 2012.
[9] C. Quinsac, A. Basarab, and D. Kouame, "Frequency domain compressive sampling for ultrasound imaging," vol. 12, pp. 1–16, 2012.
[10] M. Schiffner and G. Schmitz, "Fast pulse-echo ultrasound imaging employing compressive sensing," pp. 688–691, 2011.
[11] E. Candes and J. Romberg, "Sparsity and incoherence in compressive sampling," 2006.
[12] E. J. Candes, "The restricted isometry property and its implications for compressed sensing," *Compte Rendus de l'Academie des Sciences*, vol. 346, pp. 589 – 592, 2008.
[13] I. Tosic and P. Frossard, "Dictionary learning," *Signal Processing Magazine, IEEE*, vol. 28, no. 2, pp. 27–38, 2011.
[14] M. Aharon, M. Elad, and A. Bruckstein, "k -svd: An algorithm for designing overcomplete dictionaries for sparse representation," *Signal Processing, IEEE Transactions on*, vol. 54, no. 11, pp. 4311–4322, 2006.
[15] L. Smith and M. Elad, "Improving dictionary learning: Multiple dictionary updates and coefficient reuse," *Signal Processing Letters, IEEE*, vol. 20, no. 1, pp. 79–82, 2013.
[16] Ksvd. [Online]. Available: http://www.cs.technion.ac.il/ronrubin/software.html