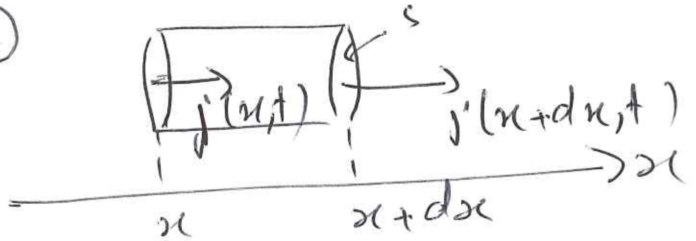
Corrigé du Partiel de thermo 2. du 04/05/17

A- Diffusion thermique

Exercice 1 [6 pts]

[1 pt] ① $\vec{j}_{th} = -\lambda \, \overrightarrow{grad}\, T$

②



$d\tau = S\,dx$

1er principe : $dU = \delta Q$; $\delta Q = dm\, c\, dT = \rho c\, d\tau\, dT$

$$\boxed{dU = \rho S dx\, c\, dT(x,t)}$$

Bilan thermique dans le volume élémentaire durant dt : $\left[j_{th}(x,t) - j_{th}(x+dx,t)\right] S\,dt = \rho S dx\, c\, dT$

[2 pts] soit $\boxed{\dfrac{\partial j_{th}(x,t)}{\partial x} = -\rho c \dfrac{\partial T(x,t)}{\partial t}}$

[1 pt] ③ $j_{th} = -\lambda \dfrac{dT}{dx}$ d'où l'éq. du bilan local d'énergie devient : $\boxed{D \dfrac{\partial^2 T(x,t)}{\partial x^2} = \dfrac{\partial T(x,t)}{\partial t} \; ; \; D = \dfrac{\lambda}{\rho c}}$

[1 pt] ④ ① $\dfrac{\partial}{\partial t} = 0$ $\quad \dfrac{\partial^2 T(x)}{\partial x^2} = 0 \quad T(x) = ax + b \quad \boxed{T(x) = \dfrac{(T_2 - T_1)}{L} x + T_1}$

[1 pt] (4.2) $j_{dif} = -\lambda \dfrac{dT}{dx} = -\lambda a \qquad \boxed{j_{dif} = \dfrac{\lambda (T_1 - T_2)}{L}}$

[7 pts] Ex 2

[1 pt] (1.1) Pas de source d'énergie $\Rightarrow \sigma = 0$

Régime stationnaire $\Rightarrow \frac{\partial}{\partial t} = 0$

(1.2) $\cancel{\Delta T = 0}$ / $\frac{\partial}{\partial r}\left(r \frac{\partial T(r)}{\partial r}\right) = 0$

$r\frac{dT}{dr} = cte = A$ $\qquad$ $T = A\ln r + B$

$T_1 = A\ln r_1 + B$ $\qquad$ $T_2 = A\ln r_2 + B$

[1,5 pt] $$\boxed{T = (-T_2 + T_1)\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_1}{r_2}\right)} + T_1}$$

[1,5 pt] (1.3) $\phi = j S = j_{th}\, 2\pi r L$; $j_{th} = -\lambda \frac{dT}{dr} \Rightarrow \boxed{\phi = \frac{(T_1 - T_2)2\pi L\lambda}{\ln \frac{r_2}{r_1}}}$

[1,5 pt] (2) $$\boxed{R_{th} = \frac{T_1 - T_2}{\phi} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L\lambda}}$$

[1,5 pt] (3) $\phi = j_c S$ et $\phi = hS(T_p - T_f) = h\, 2\pi r_2 L(T_p - T_f)$

$\Delta T = T_p - T_f = R_c \phi$ $\qquad$ d'où $\boxed{R_c = \frac{1}{h 2\pi r_2 L}}$

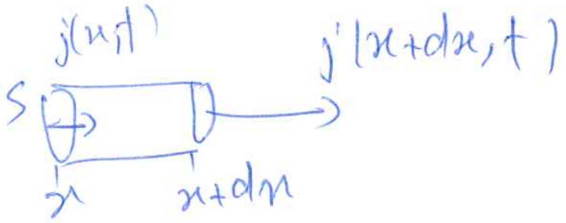
B- Diffusion de particules

|5pts|

Exercice 3

|1pt| ① $j = -D \dfrac{\partial n(x,t)}{\partial x}$ (loi de Fick) D en $m^2 s^{-1}$

②



$dN = j(x,t)\,dS\,dt - j(x+dx,t)\,dS\,dt$

$dn(x,t) = \dfrac{dN}{S\,dx} = \left[j(x,t) - j(x+dx,t)\right]dt = -\dfrac{\partial j(x,t)}{\partial x}\,dt$

|1pt| La démonstration n'est pas demandée

d'où $\boxed{\dfrac{\partial j(x,t)}{\partial x} + \dfrac{\partial n(x,t)}{\partial t} = 0}$

d'après ① $-D\,\partial^2 n(x,t)$

Éq. de conservation des particules

|1pt| ③ La loi de Fick dans l'Éq. de conservation implique :

$\boxed{\dfrac{\partial n(x,t)}{\partial t} = D\dfrac{\partial^2 n(x,t)}{\partial x^2}}$ Éq. de diffusion des particules

3 pts

4

$$* \frac{\partial n}{\partial t} = \frac{N_0}{S\sqrt{4\pi D}} \left(\frac{-1}{2t\sqrt{t}} \exp \frac{-x^2}{4Dt} \quad \frac{-x}{2Dt\sqrt{t}} \exp \frac{-x^2}{4Dt} \right) \quad (a)$$

$$* \frac{\partial n}{\partial x} = \frac{N_0}{S\sqrt{4\pi Dt}} \left(\frac{-2x}{4Dt} \right) \exp \frac{-x^2}{4Dt}$$

$$* \frac{\partial^2 n}{\partial x^2} = \frac{-N_0}{2S\sqrt{4\pi Dt} \cdot Dt} \left(\exp \frac{-x^2}{4Dt} - 2x^2 \exp \frac{-x^2}{4Dt} \right) \quad (b)$$

(a) = D(b) d'où $n(x,t)$ est solution de l'éq. de diffusion.

3 pts Exercice 4

3 pt

1 $\quad D \frac{\partial^2 n(x,t)}{\partial x^2} = \frac{\partial n(x,t)}{\partial t}$

$\frac{\partial}{\partial t} = 0 \qquad n(x,t) = Ax + B$

$n_1 = B$

$n_2 = AL + B$

$\Rightarrow A = \frac{n_2 - n_1}{L}$

$A = -2{,}2 \; 10^{22} \; m^{-4}$

1,5 pt

$$\boxed{n(x,t) = \frac{(n_2 - n_1)}{L} x + n_1}$$

2 $\quad j = -D \frac{\partial n}{\partial x} = -DA$

$$D = \frac{-j}{A}$$

1,5 pt

$$\boxed{D = \frac{jL}{n_1 - n_2}}$$